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Estimating and Forecasting Tax Revenues Using GARCH Family of Models: A Case of Pakistan

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ABSTRACT

Forecasting plays a vital role in effective planning and decision-making for policy formulation across a variety of fields of life. The Nonlinear models such as the GARCH family, including symmetric and asymmetric generalized autoregressive conditional heteroscedastic (GARCH) models with both normal and non-normal innovations are applied in this study to capture the dynamic and asymmetric features of the two tax revenue series, sales Tax and Direct Tax in Pakistan. Additionally, Autoregressive Moving Average (ARMA) model is used as the mean model. The prime objective of this research is to examine the estimating and forecasting performance of ARMA models along GARCH family models, for the monthly tax revenue series in Pakistan, particularly focusing on symmetric GARCH and asymmetric GARCH models (EGARCH, TGARCH, and PARCH). Empirical evidence based on the application of these models to the selected series reveals that the GARCH family models effectively confine the heteroscedasticity, highlighting the strength of these models. In addition, three distributions, normal, Student-t and generalized error distribution are considered for the residuals. Under the normal distribution, ARMA(5,4)-EGARCH(1,1,2) model is selected as the best model based on minimum values of MAE, RMSE, MAPE, and TIC. In the same way, for the Student-t and Generalized Error Distribution, the ARMA(5,4)-EGARCH(1,1,1) and ARMA(5,4)-EGARCH(1,1,3) models are selected as the best forecasting models for the sales tax series. The ARMA(3,3)-EGARCH(1,1,1) model is selected as the best forecasting model based on the minimum values of MAE, RMSE, MAPE, and TIC for the direct tax series assuming a normal distribution. Similarly, the ARMA(3,3)-TGARCH(1,1,1) and ARMA(3,3)-TGARCH(1,1,1) models are selected as the best forecasting models for the direct tax.

Introduction

Taxes play a significant role in the functioning of governments and economies as provide the primary source of revenue for maintaining infrastructure and public services. These play a pivotal role in economic stability and growth, reducing inequality and wealth redistribution. Taxes support long-term sustainability and empower governments to cater to the diverse needs of their citizens and drive overall societal well-being, by ensuring fiscal responsibility and effective government debt management.

Tax collection by the FBR in FY2022 showed a growth of 9.92% compared to the year in FY2021. Governments across the globe often employ tax expenditures (TE) as an alternative policy tool to pursue social objectives and stimulate economic growth. Nevertheless, the effective utilization and efficient administration of tax expenditure policies are crucial for attaining these desired goals. Tax serves as a fundamental revenue source for governments worldwide, constituting approximately 50 percent of their funds. The tax-to-GDP ratio is commonly used as a valuable measure for comparing tax systems among countries, as it establishes a relationship between tax revenues and the overall size of the economy (See Asif et al. 2021).

Pakistan experienced an improvement in its tax-to-GDP ratio from 2021 to 2022, reaching 9.2 percent. However, when compared to other regional countries, Pakistan's tax-to-GDP ratio remains relatively low. Over the years, the tax-to-GDP ratio in Pakistan has gradually grown since 1950, starting at 4.4 percent. It is projected to reach 9.3 percent in 2022, with further growth expected in 2023. Among the various types of taxes, two prominent categories are sales tax and direct tax. Sales tax refers to the levy imposed on the purchase of goods and services, while direct tax encompasses taxes imposed directly on individuals and entities based on their income, assets, or transactions. These two tax types hold significant importance in the overall tax landscape, each serving specific purposes and contributing to the funding of public goods, economic stability and societal welfare.

Sales tax is usually assessed as a percentage of the retail price at the time of sale. In Pakistan, the government replaced the Sales Tax with a value-added tax (VAT) in the late 1980s. The Sales Tax Act 1990 replaced the Sales Tax Act 1951 on November 1, 1990. VAT is imposed in addition to sales tax on services and products. According to Global (2023), the federal government of Pakistan raised the general sales tax to 18 percent in February 2023. A 25 percent higher tax is imposed on imported goods for subsequent supply. These products include shampoo, cigarettes, cosmetics, tissue paper and crockery.

Direct tax, on the other hand, is paid directly to the government by individuals and includes taxes such as income tax, land tax and personal property tax. It is charged based on the income of the person, with higher income individuals paying more tax. Direct tax encourages economic elasticity and equality, serving as a just and fair means to increase government revenue and reduce income disparity. However, in Pakistan, the direct tax system still needs to be fully utilized and revised. In the fiscal year 2020, the tax-to-GDP ratio in Pakistan was 8.5 percent, which is below the average of 28 percent for Asia. It is also below the standard of the OECD by 23.2 percentage points, as the average OECD tax-to-GDP ratio is 33.5 percent. Pakistan aims to collect 5.8 trillion rupees in tax revenues in the fiscal year 2021-2022, with an expected collection of up to 1.1 trillion in the current year. During the eleven months, tax receipts increased to 4.3 million and reached 4.7 million by the end of the year (See Asif et al. 2021; Qasim, Javed and Ali, 2022). In the fiscal year, the share of sales and direct taxes increased, with sales tax rising by forty-one percent and direct tax by thirty-seven percent. The percentage of direct tax contribution increased compared to indirect taxes. In 1952, indirect tax exceeded direct tax, but in 2022, direct tax contributed 37 percent while the share of indirect tax dropped from 86 percent to 62.8 percent. The collection of direct tax has shown significant growth over the years. (See Anjum et al. 2022).

According to Gleen (2000) forecasting tax revenues is essential for the government, as it ensures stability in taxes, a healthy fiscal situation, and fairness in the taxation system. It also facilitates efficient economic decision-making and financial reporting. Fiscal forecasting plays a crucial role in providing the government with the necessary financial resources to provide goods and services to its citizens. The budgetary forecast includes revenues and expenses, and a quantitative approach is used to project tax collections (Rehman et al., 2022; Sibt-e-Ali et al., 20211a,b).

In our study, we aim to identify the time series model that provides better forecasts for monthly tax revenue in Pakistan. For this purpose, ARMA model is applied as the mean model. The tax revenue series are generally subject to exhibit variation and non-constant variances due to explicit variables. To tackle these variations, GARCH models are applied with both normal and non-normal innovations. When dealing with distributions that are non-normal, the statistical methods of the Student's t-distribution and the generalized error distribution are utilized. This process is repeated for asymmetric GARCH models including GJR GARCH, EGARCH and PARCH models. To conclude forecasting results from the selected models are compared through the evaluation of the series. The main motivation for this study is to apply the nonlinear models to capture the volatile feature of the tax time series in Pakistan.

Literature Review

Streimikiene et al. (2018) studied Pakistan's monthly tax income projected using three distinct time series methods: an autoregressive model with seasonal dummies, a VAR and an autoregressive integrated moving average (ARIMA) to predict Pakistan's tax revenue. Their results showed that ARIMA model is the most effective and significant. Alhassan et al. (2022) studied the GARCH family and vector auto regression to analyze the tax revenue behavior in Nigeria. Their results discovered that tax revenue behavior is typically impacted by previous tax revenue data. Moreover, their study showed that the tax revenue volatility from the preceding month may be harmful to the current tax revenue volatility. Arif et al. (2014) analyzed and constructed time series forecast models for Pakistan's indirect and direct tax revenue, encompassing a span of 40 years from 1973 to 2011. The best forecast models for indirect federal revenue and direct income tax were found in the ARIMA (1,1,1) and ARIMA (1,1,0) models. Chaudhry et al. (2010) examined the reasons for Pakistan's poor tax revenue using time series econometric methods from 1973 to 2009. Their empirical results demonstrated the major influence of exogenous variables on the projected consequences of Pakistan's tax efforts. This study also found that low tax collection in Pakistan is due to the country's limited tax base, dependence on foreign aid, low literacy rates and reliance on agriculture. Gerardo et al. (2014) employed both static and dynamic panel data approaches to study how economic, institutional and social factors impact tax collection in 34 OECD countries. Their results showed that both the agricultural area and the proportion of foreign direct investment in gross fixed capital formation had adverse impacts on the dependent variable.

Abhijit et al. (2007) utilized a huge dataset to promote the existing empirical research concerning the significant factors that affect tax revenue performance in rising countries. The results confirmed that certain structural elements, such as per capita GDP, the role of agriculture in trade openness, GDP and foreign aid have a substantial impact on an economy's ability to generate revenue. Additionally, the study revealed that factors like corruption levels, political stability, and the proportion of direct and indirect taxes also influenced the performance of tax revenue.

Data and Methodology

In the current study, we analyzed the monthly sales tax and direct tax revenue of Pakistan. The monthly data are taken for the period 2003-2021 and are obtained from the website fbr.gov.pk. The dataset contains 228 observations spanning from January 2003 to December 2021. Out of these, 192 observations are used for estimation, covering the period from January 2003 to December 2018, while the remaining 36 observations, and used for evaluating the forecast. While proceeding with this study, we apply the GARCH family of models (ARMA-GARCH, TGARCH, EGARCH, and PARCH) to handle the problem of heteroscedasticity by using three distributions Normal, Student-t and GED.

Models

Autoregressive Moving Average (ARMA) Model

The Autoregressive Moving Average model, denoted as ARMA (l, m), combines both autoregressive and moving average models. An ARMA model with l order for the autoregressive (AR) component and m order for the moving average (MA) component is used when a series exhibits both AR and MA characteristics. The equation of ARMA (l, m) model is given as:

$$Zt = \phi 0 + \phi 1 Zt-1 + \phi 2 Zt_{-2} + \ldots + \phi_l Zt-l + \omega_1 \epsilon t_{-1} + \omega_2 \epsilon_{t-2} + \cdots + \omega q \epsilon_{t-m} + \epsilon_t$$

where $\epsilon_t \sim IN\,(0, \sigma^2)$

Family of GARCH Models

The Generalized Autoregressive Conditional Heteroscedastic model by Bollevsleve (1986), an extension of ARCH model by Engle (1982) is mathematically given as:

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{p} \beta_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \eta_j \sigma_{t-j}^2$$

β_0 is a constant term, β_1 , β_1 , … . β_P ranges from 0 to 1 are the parameters of the GARCH specification $\eta_{1,} \eta_{2, \ldots ,} \eta_q$ also ranging from 0 to 1.

EGARCH Model

The exponential GARCH (EGARCH) model, introduced by Nelson (1991), is the first asymmetric volatility model to accommodate the leverage effect. The specifications of this model are as under:

$$ln\sigma_t^2 = \beta_0 + \sum_{i=1}^{p} (\beta_i \left|\frac{\epsilon_{t-i}}{\sigma_{t-i}}\right| + \gamma_i \frac{\epsilon_{t-i}}{\sigma_{t-i}}) + \sum_{j=1}^{q} \eta_j ln\sigma_{t-j}^2$$

TGARCH Models

The TGARCH model, by Gloston, Jagannathan and Runkle (1993)), is given as:

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{p} \beta_i \; {}^{2}_{t-i} + \sum_{j=1}^{q} \eta_j \sigma_{t-j}^2 + \sum_{k=1}^{r} \gamma_k \; {}^{2}_{t-k} \, \tau_{t-k}$$

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{p} (\gamma_i (1 - I_{t-i}) \epsilon_{t-i}^2 + \beta_i I_{t-i} \epsilon_{t-i}^2) + \sum_{j=1}^{q} \eta_j \sigma_{t-j}^2$$

Where I_t = 1 if ϵ_t <0 and 0 otherwise

PARCH Model

Ding et al. (1993) proposed APARCH model. The specifications of this model are:

$$\sigma_t^\delta = \beta_0 + \sum_{i=1}^{s} \beta_i \, (|\epsilon_{t-i}| - \gamma_i \epsilon_{t-i})^\delta + \sum_{j=1}^{q} \eta_j \sigma_{t-j}^\delta$$

Distributions

We have considered three distributions in this study.

Normal Distribution

The normal distribution, also known as the Gaussian or bell curve, is a smooth probability distribution characterized by its symmetry. It is defined by two parameters: the mean and standard deviation.

$$\boldsymbol{f(\zeta_t) = \frac{1}{\sqrt{2\pi}} exp(-\frac{\zeta_t^2}{2})}$$

where $\zeta_t = \frac{\epsilon_t}{\sigma_t}$

Student-t Distribution

Bollerslev(1987) Student-t Distribution, with GARCH models. Which is specified as

$$f(\zeta_t) = \frac{\Gamma\left(\frac{1+\eta}{2}\right)}{\Gamma(\frac{\eta}{2})(\sqrt{\pi(\eta-2)}} (1 + \frac{\zeta_t^2}{\eta - 2})^{-\frac{\eta+1}{2}}$$

The Generalized-Error Distribution (GED)

The pdf of the generalized error distribution is given as

$$f(\zeta_t) = \frac{\eta exp(-0.5\left|\frac{\zeta_t}{\lambda}\right|^{\eta})}{2^{(1+\frac{1}{\eta})}\Gamma(\eta^{-1})\lambda}$$

Where η is the shape parameter and $\lambda = \left[2^{-\frac{2}{\eta}}\Gamma(\frac{1}{\eta})/\Gamma(\frac{3}{\eta})\right]^{1/2}$

Result and Discussion

The plots of all the series at level are given in Figure 1 showing the non-stationarity of the data. Figure 2 displays the first log difference series showing a stationary pattern for each series. For checking the stationarity of the data sets, the Augmented Dickey-Fuller (ADF) unit root test is also applied and the results are described in Table 1. It is clear that both the series are non-stationary at the level and stationary at the first log difference

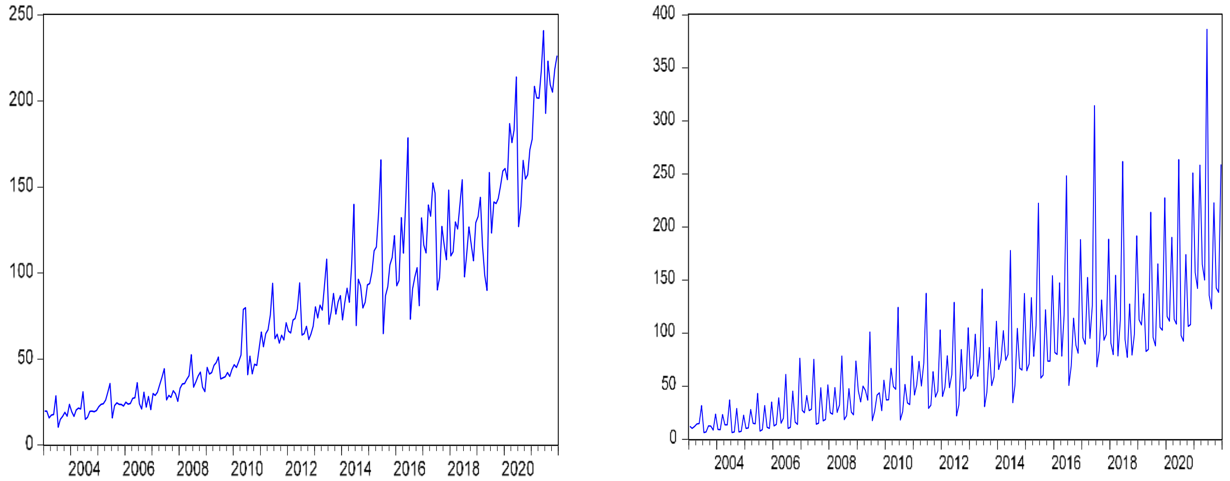

Figure 1: Monthly Tax Revenues Series

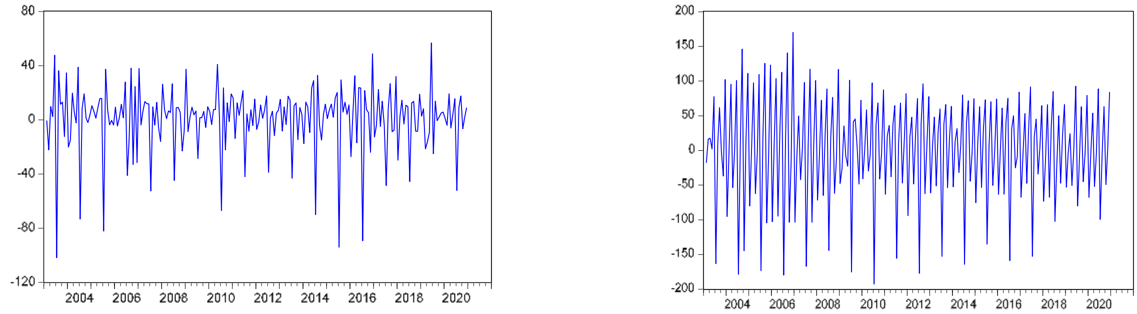

Figure 2: Plots of log difference Series of Tax Revenues Series

Table 1: Unit Root Test for Tax Revenues

Series	Level		After log difference	
	t-statistic	p-value	t-statistic	p-value

Sale Tax	2.899461	1.00000	-9.916625	0.0000
Direct Tax	1.893956	0.9998	-14.0022	0.0000

The descriptive statistics of the Tax Revenue series are shown in Table 2. These results indicate that the distribution of each series is not normal for both at the level and at first log difference. We have applied ARMA models with a suitable order and observed the correlogram of ACF and PACF of the residual and squared residuals of these models. The selected best ARMA model for each series is given in Table 3.

Table 2: Descriptive Statistics of Tax Revenue Series

Series at level	**Mean**	**Median**	**Std. Dev**	**Skewness**	**Kurtosis**	**Jarque-bera**	**p-value**
Sale Tax	80.947	70.550	54.73	0.808	2.874	24.980	0.0004
Direct Tax	76.046	61.100	63.379	1.5930	6.326	201.5876	0.000
After Achieving Stationarity							
Sale Tax	0.8955	0.8384	10.520	0.35370	8.6363	305.204	0.0000
Direct Tax	0.591	-0.469	17.445	1.404	11.277	722.6	0.0000

Table 3: Selected ARMA Models for Tax Revenue Series

Series	**Model**	**AIC**
Sale Tax	ARMA(5,4)	6.536579
Direct Tax	ARMA(3,3)	14.234455

During the diagnostics checking the correlogram of squared residual, it is found that the ACF and PACF do not remain within the critical limits indicating the existence of conditionally heteroscedastic. (Results are not presented here). We have applied GARCH models to model this. Subsequently, we estimated different ARMA models with GARCH orders and observed the Correlogram of the residual and squared residual of these models to identify the GARCH orders. Only GARCH (1, 1) model fulfills the diagnostics measures.

Table 4: Estimated Results for ARMA (5, 4)-GARCH (1, 1) Model for Sale Tax Series

Coefficient	**Estimate**	**S.E**	**Z-Statistic**	**P-Value**
Φ_0	0.409128	0.093157	4.391801	0.0000
Φ_1	0.220726	0.337026	0.654922	0.5125
Φ_2	-0.549058	0.218595	-2.511766	0.0120
Φ_3	0.234700	0.309122	0.759246	0.4477
Φ_4	-0.198286	0.105476	-1.879920	0.0601
Φ_5	-0.264046	0.103305	-2.555987	0.0106
ω_1	-0.963885	0.343913	-2.802698	0.0051
ω_2	0.785899	0.392382	2.002892	0.0452
ω_3	-0.762676	0.402278	-1.895893	0.0580
ω_4	0.357158	0.273933	1.303816	0.1923

Variance Equation				
β_0	-0.138019	0.048584	-2.840823	0.0045
β_1	-0.029907	0.005242	-5.705639	0.0000
η_1	0.053941	7.80E-05	13509.15	0.0000

Table 4 shows the estimated results of the selected model under a normal equation for sales Tax series. In the mean equation, the coefficients Φ_1, Φ_3, Φ_4 and ω_4 are insignificant and the others are highly significant. In the variance equation the coefficients β_0, β_1 and η_j are significant.

Table 5: Estimated Results for ARMA(3,3)- GARCH (1,1) Model for Direct Tax Series

Coefficient	Estimate	S.E	Z-Statistic	P-Value
Φ0	0.369705	0.106271	3.478876	0.0005
Φ1	-0.264032	0.247480	-1.066883	0.2860
Φ2	-0.342516	0.232839	-1.471041	0.1413
Φ3	0.122603	0.107337	1.142223	0.2534
$\omega 1$	-0.534668	0.238842	-2.238586	0.0252
$\omega 2$	0.096456	0.182831	0.527567	0.5978
$\omega 3$	-0.311330	0.180496	-1.724852	0.0846
Variance Equation				
β_0	0.052846	0.092531	0.571114	0.5679
β_1	0.034343	0.005792	5.929444	0.0000
η_1	0.052325	0.000132	7954.210	0.0000

We have also applied ARMA-GARCH model with Student-t and GED. Moreover, asymmetric GARCH models such as EGARCH, TGARCH and PARCH with Normal, Student-t and GED distribution are fitted. The estimated results of the selected models for all the distributions are presented below.

Table 6: Estimated Results of ARMA(5,4) - TGARCH(1, 1, 1) Model with Normal Distribution for Sales Tax Series

Coefficient	Estimate	S.E	Z-Statistic	P-Value
Φ_0	0.472620	0.035346	13.37143	0.0000
Φ_1	0.223030	0.227500	0.980351	0.3269
Φ_2	-0.262425	0.272594	-0.962699	0.3357
Φ_3	0.030782	0.254226	0.121080	0.9036
Φ_4	-0.180262	0.100521	-1.793278	0.0729
Φ_5	-0.320135	0.090197	-3.549289	0.0004
ω_1	-0.965675	0.248804	-3.881271	0.0001
ω_2	0.573461	0.432829	1.324912	0.1852
ω_3	-0.368977	0.457238	-0.806970	0.4197
ω_4	0.206011	0.254465	0.809582	0.4182
Variance Equation				
β_0	0.551262	0.107607	5.122934	0.0000
β_1	0.041351	0.008847	4.673814	0.0000
γ_1	-0.214736	0.002434	-88.24100	0.0000
η_1	0.041446	0.000556	1873.049	0.0000

Table 6 shows the estimated results of TGARCH (1, 1, 1) model for normal distribution for the Direct Tax series. In the mean equation of the model, the coefficients Φ_0, Φ_5 and ω_1 are significant and the other are insignificant. In the variance equation, all coefficients are significant.

Table 7: Estimated Results of ARMA(5,4)-EGARCH(1, 1, 1) Model with Student-t Distribution for Sales Tax Series

Coefficient	Estimate	S.E	Z-Statistic	P-Value
Φ_0	0.433038	0.073731	5.873240	0.0000
Φ_1	-0.066848	0.335759	-0.199095	0.8422
Φ_2	-0.435420	0.244240	-1.782759	0.0746
Φ_3	0.081083	0.329366	0.246178	0.8055
Φ_4	-0.149738	0.109776	-1.364029	0.1726
Φ_5	-0.248201	0.095799	-2.590851	0.0096
ω_1	-0.723428	0.353659	-2.045549	0.0408
ω_2	0.478527	0.407709	1.173697	0.2405
ω_3	-0.513575	0.415484	-1.236089	0.2164
ω_4	0.192258	0.303309	0.633868	0.5262
Variance Equation				
β_0	0.081919	0.043416	1.886855	0.0592
β_1	0.085629	0.053385	1.604005	0.1087
β_2	0.205956	0.138482	1.487235	0.1370
γ_1	-0.171595	0.142849	-1.201237	0.2297
η_1	0.998937	1.8E-104	5.6E+103	0.0000

The results of the estimated EGARCH (1, 1, 1) model with student-t distribution are presented in Table 7. These results indicate that the coefficients Φ_1, Φ_3 and ω_4 are insignificant and the other is significant. In the variance equation β_0, γ_1 are insignificant η_1 and β_1 are significant. In the distribution, the degree of freedom coefficient η is significant indicating the justification of the use of Student-t distribution.

Table 8: Estimated Results of ARMA(5,4)-TGARCH(1, 2, 1) Model with GED for Sales Tax Series

Coefficient	Estimate	S.E	Z-Statistic	P-Value
Φ_0	0.493845	0.040931	12.06534	0.0000
Φ_1	0.063167	0.288956	0.218604	0.8270
Φ_2	-0.003001	0.317704	-0.009446	0.9925
Φ_3	0.002849	0.288644	0.009870	0.9921
Φ_4	-0.234644	0.121099	-1.937630	0.0527
Φ_5	-0.268260	0.082702	-3.243699	0.0012
ω_1	-0.703500	0.305817	-2.300394	0.0214
ω_2	0.101918	0.447286	0.227859	0.8198
ω_3	-0.117195	0.406767	-0.288113	0.7733
ω_4	0.244167	0.248591	0.982205	0.3260

Variance Equation				
β_0	0.524074	0.154128	3.400247	0.0007
β_1	0.060084	0.019957	3.010676	0.0026
γ_1	-0.197021	0.002023	-97.39162	0.0000
γ_2	-0.051773	0.043212	-1.198109	0.2309
η_1	0.045126	0.000520	2008.901	0.0000
Distribution Coefficient				
η	1.390108	0.653276	2.127903	0.0333

Table 8 presents the estimated results of the TGARCH (1, 2, 1) model with Generalized Error distribution. The coefficients Φ_0, Φ_4, Φ_5 and ω_1 are significant while the other coefficient is insignificant. In the variance equation β_0, β_1, γ_1 and $\eta_1$are significant and γ_1 is insignificant. The thickness coefficient η is also significant.

Table 9: Estimated Results of ARMA(3,3) - EGARCH(1,1,1) Model with Normal Distribution for Direct Tax Series

Coefficient	Estimate	S.E	Z-Statistic	P-Value
$\Phi 0$	0.270530	0.624645	0.433094	0.6649
$\Phi 1$	-0.294955	0.212069	-1.390840	0.1643
$\Phi 2$	-0.204328	0.172979	-1.181231	0.2375
$\Phi 3$	0.038432	0.092867	0.413843	0.6790
$\omega 1$	-731980.9	56321.23	-12.99654	0.0000
$\omega 2$	129718.6	166310.0	0.779980	0.4354
$\omega 3$	-13236.11	122773.6	-0.107809	0.9141
Variance Equation				
$\beta 0$	29.37577	3.443186	-8.531567	0.0000
$\beta 1$	0.505693	0.041889	-12.07233	0.0000
$\gamma 1$	-0.465214	0.033406	-13.92621	0.0000
$\eta 1$	-0.319647	0.150040	-2.130409	0.0331

The estimated results of the EGARCH model with normal distribution are given in Table 9. The empirical evidence leads to the conclusion that the coefficients Φ_0, Φ_1, Φ_2, Φ_3 and ω_3 are insignificant and the other coefficients are significant. In the variance equation, all coefficients are significant justifying the use of asymmetric model to capture the possible leverage effect.

Table 10: Estimated Results of ARMA(3, 3)- EGARCH(1,1,1) Model with Students-t Distribution for Direct Tax Series

Coefficient	Estimate	S.E	Z-Statistic	P-Value
Φ_0	0.327458	0.088075	3.717961	0.0002
Φ_1	-0.496764	0.203321	-2.443250	0.0146
Φ_2	-0.487408	0.204169	-2.387280	0.0170

Φ_3	0.129828	0.095456	1.360082	0.1738
$\square_1$	-0.271581	0.190882	-1.422774	0.1548
$\square_2$	0.049775	0.120495	0.413093	0.6795
$\square_3$	-0.421364	0.142601	-2.954840	0.0031
Variance Equation				
$\square_0$	0.085610	0.040295	2.124577	0.0336
$\square_1$	0.000934	0.020817	0.044877	0.9642
$\square_1$	-0.083286	0.049812	-1.672015	0.0945
$\square_1$	0.999868	1.7E-104	5.8E+103	0.0000
Distribution Coefficient				
η	7.210167	4.129946	1.745826	0.0808

Table 10 highlights the estimated results for EGARCH (1, 1, 1) model with Student-t distribution. In the mean equation, the coefficients Φ_0, $\Phi 1$, Φ_2, and $\square_3$ are significant and the other coefficients are insignificant. In the variance equation β_0 and $\square_1$ are significant and the other coefficients are insignificant. The degree of freedom parameter is also significant indicating that the techniques applied are appropriate to capture the hidden feature of the data.

Table 11: Estimated Results of ARMA(3,3)- EGARCH(1,1,1) Model with Generalized Error Distribution for Direct Tax Series

Coefficient	Estimate	S.E	Z-Statistic	P-Value
Φ_0	0.224615	0.095646	2.348401	0.0189
Φ_1	-0.485445	0.176670	-2.747749	0.0060
Φ_2	-0.423472	0.190452	-2.223511	0.0262
Φ_3	0.231408	0.093503	2.474867	0.0133
$\square_1$	-0.222570	0.172686	-1.288870	0.1974
$\square_2$	0.048003	0.111713	0.429704	0.6674
ω_3	-0.454908	0.122274	-3.720409	0.0002
Variance Equation				
$\square_0$	0.019516	0.054598	0.357441	0.7208
$\square_1$	0.049904	0.063405	0.787065	0.4312
$\square_1$	-0.000918	0.060634	-0.015145	0.9879
η_1	0.000706	0.005723	174.8567	0.0000
Distribution Coefficient				
η	1.068931	0.143507	7.448617	0.0000

Table 11 identifies the estimated results of EGARCH (1, 1, 1) model with the Generalized Error distributions for Direct Tax series. These results depict that in the mean equation, the coefficients Φ_0, Φ_1, Φ_2, Φ_3, and ω_3, are significant while the other coefficients are insignificant while in the variance equation, η_1 is significant and the others are insignificant. The shape coefficient η is also significant.

Forecasting Evaluation

Forecast evaluation is the process of assessing the accuracy and effectiveness of a forecasting model. The goal of forecast evaluation is to determine whether a particular model is useful for making future predictions and if it can be relied upon to provide accurate forecasts. In this study, a comparative evaluation of the forecasting accuracy for the selected ARMA-GARCH-type models for all distributions (Nornal, student t, and GED) is assessed. The results based on loss functions are presented in Table 12.

Table 12: Forecast Evaluation for Sale Tax Series

	Models	MAE	RMSE	MAPE	TIC
Normal Distribution	ARMA(5,4)-TGARCH(1,1,1)	14.00749	19.30678	100.5885	0.962017
	ARMA(5,4)-EGARCH(1,1,2)	13.89245	19.12053	102.2924	0.952852
	ARMA(5,4)-PARCH(1,1,1)	13.89069	19.14566	102.8360	0.954347
Student-t Distribution	ARMA(5,4)-TGARCH(1,1,3)	13.99515	19.27169	101.1833	0.963660
	ARMA(5,4)-EGARCH(1,1,1)	13.87110	19.09519	104.2080	0.952444
	ARMA(5,4)-PARCH(1,1,1)	13.93633	19.16341	102.7780	0.960633
Generalized Error Distribution	ARMA(5,4)-TGARCH(1,1,2)	13.98136	19.21436	101.7929	0.962351
	ARMA(5,4)-EGARCH(1,1,3)	13.94411	19.14581	102.4954	0.960128
	ARMA(5,4)-PARCH(1,1,1)	13.96043	19.21707	101.7496	0.959970

A look at these results leads to conclude that for the normal distribution, the ARMA(5,4)-EGARCH(1,1,2) provide the best forecasting ability based on the minimum value of MAE, RMSE, MAPE and TIC. Similarly for Student t and Generalized Error Distribution the ARMA(5,4)-EGARCH(1,1,1) and ARMA(5,4)-EGARCH(1,1,3) models provide the best forecasting performance for the sale tax series.

Table 13: Forecast Evaluation for Direct Tax Series

	Models	MAE	RMSE	MAPE	TIC
Normal Distribution	ARMA(3,3)-TGARCH(1,1,1)	21.29843	29.92727	173.5552	0.704321
	ARMA(3,3) EGARCH(1,1,1)	19.48291	27.93409	152.4616	0.644114
	ARMA(3,3)-PARCH(1,1,1)	20.83549	29.90251	125.5992	0.727743
Student-t Distribution	ARMA(3,3)-TGARCH(1,1,1)	20.49387	29.13012	150.1902	0.697713
	ARMA(3,3)-EGARCH(1,1,1)	20.69634	29.74139	120.3144	0.732341

	ARMA(3,3)-PARCH(1,1,1)	20.65710	29.79276	123.4375	0.735161
Generalized Error Distribution	ARMA(3,3)-TGARCH(1,1,1)	20.00759	28.90347	143.9397	0.703597
	ARMA(3,3)-EGARCH(1,1,1)	20.01951	29.12116	144.0104	0.728974
	ARMA(3,3)-PARCH(1,1,1)	20.06872	29.45414	118.5621	0.746974

The results of the forecast evaluation of selected models under normal distribution, Student-t and Generalized-Error Distribution for Direct Tax series are presented in Table 13. It is obvious that for the normal distribution, ARMA(3,3)-EGARCH(1,1,1) performs the best Similarly for Student-t and Generalized-Error Distribution the ARMA(3,3)-TGARCH(1,1,1) seems to perform the best for the Direct tax series.

Conclusion and Recommendation

Modeling and forecasting sales tax and Direct tax time series are crucial for effective fiscal planning and budget management. Reliable tax forecasts enable governments to optimize revenue collection, control shortfalls, allocate resources and make effective policy decisions. The GARCH family of models is a powerful tool to provide reliable forecasts. These models are flexible to capture the dynamics and asymmetry in shocks which are common in financial time series. This study compares the GARCH family of models including GARCH, EGARCH, TGARCH and PARCH models with three distributional assumptions, normal, Student-t and GED to capture variations asymmetries and fat-tailed nature of the financial time series. The proposed methodology is applied to two Tax series(sale tax and Direct Tax) in Pakistan. The empirical results in both series support the use of ARIMA modeling along with GARCH family of models. The statistical significance of the majority of the coefficients supporting the techniques applied. The asymmetric coefficient in the asymmetric GARCH models is significant for both the series justifying the application of asymmetric GARCH models. Moreover, the shape parameter in the non-normal distribution for all the cases for both the series is statistically significant depicting the power of capturing heavy tailed property of the data under consideration.

Based on this study, it is recommended that future research in this domain should explore advanced modeling techniques such as regime switching models, and machine learning method to improve forecasting accuracy and handle the complex non-linear patterns in tax data in Pakistan. To identify the potential tax evasion, fraud and noncompliance risks, utilize advanced data analytics techniques. The robust data management systems and building analytical capabilities within tax administration can significantly contribute to revenue enhancement and investment. The government should concentrate on expanding the Tax base by bringing more individuals and businesses into the tax network. Moreover, regulations and filing procedures should be improved which makes the tax system more transparent and easier to comply with Streamlining and simplifying tax laws. To reduce the burden of compliance and minimize the scope for interpretation, tax guidance should be provided to taxpayers.

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